Basic Feedback and its various types

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The fundamental idea behind feedback control is to **modify the stability characteristic** of given system which has unsatisfactory inherent stability behavior.

For example if a system has insufficient damping a feedback may arranged in order to improve the damping.

As a general rule, any one of the following feedback may be used to improve the damping

- Position feedback (also called stiffness feedback)
- Velocity feedback (also called rate feedback)
- Acceleration feedback
Example of Spring – Mass – Damper system
The equation of motion for the system

\[ m\ddot{x} + c\dot{x} + kx = f(t) \]

Taking Laplace transform for the initial condition yields

\[ ms^2x(s) + csx(s) + kx(s) = f(s) \]

\[ (ms^2 + cs + k)x(s) = f(s) \]

\[ \frac{x(s)}{f(s)} = \left( \frac{1}{ms^2 + cs + k} \right) \]

The dynamic stability behaviour of the system is determined completely by the roots of its characteristic equation also called "system equation".

\[ ms^2 + cs + k = 0 \]
The roots of this equation taking the following form

\[ s_{1, 2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m} \]

Note: The system will be stable as long as \( C > 0 \).
Block diagram of the system with three feedback loops
\[ m\ddot{x} + c\dot{x} + kx = f(t) \]

*The above equation of motion now modified as (because of feedback)*

\[ m\ddot{x} + c\dot{x} + kx = f(t) - k_1x - k_2\dot{x} - k_3\ddot{x} \]

*Taking the laplace transformation for zero initial condition*

\[ \left\{ (m + k_3)s^2 + (c + k_2)s + (k + k_1) \right\} x(s) = f(s) \]

\[ \frac{x(s)}{f(s)} = \left( \frac{1}{(m + k_3)s^2 + (c + k_2)s + (k + k_1)} \right) \]

*The characteristic equation for the feedback system is*

\[ (m + k_3)s^2 + (c + k_2)s + (k + k_1) = 0 \]
The roots of this characteristic equation may be expressed as

\[ s_{1,2} = \frac{-(c + k_2) \pm \sqrt{(c + k_2)^2 - 4(m + k_3)(k + k_1)}}{2(m + k_3)} \]

- The above equation is said to be “augmented system equation”. From the above equation it is clear that the role of the feedback gain \( k_1, k_2, k_3 \) is to alter the root in the S-plane.

- The effect of varying the position, velocity and acceleration feedback gains are shown in the figure.
For open loop system:
- $m = 5$ slugs
- $c = 50$ lbs/ft/sec
- $k = 250$ lbs/ft

For open loop system:
- $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{50} = 7.07$ rad/sec
- $\zeta = \frac{c}{2\sqrt{km}} = 0.707$

Root loci for: $m\ddot{x} + c\dot{x} + kx = f(t) - k_1\dot{x} - k_2\ddot{x} - k_3\dddot{x}$

Position feedback locus
- $k_1 \neq 0$, $k_2 = k_3 = 0$

Velocity feedback locus
- $k_2 \neq 0$, $k_1 = k_3 = 0$

Acceleration feedback locus
- $k_3 \neq 0$, $k_1 = k_2 = 0$

Root locus for Position—Velocity and Acceleration feedback
Position Feedback

- Position feedback $k_1$ affects both the undamped natural frequency and damping ratio of the closed loop system.

- The root locus of the case is a straight line which passes through the open loop system poles.

- It is noted that the real part of the closed loop system roots remains constant as $k_1$ is varied while $k_2$ and $k_3$ are kept at zero.
Velocity feedback

- Velocity feedback or rate feedback $k_2$ affects the damping ratio of the closed loop system only.

- The root locus in the case is a circular around the origin of the S-plane.

- It is noted that the undamped natural frequency of the closed loop system remains constant as $k_2$ is varied while $k_1$ and $k_3$ are kept at zero.
ACCELERATION FEEDBACK

- Acceleration feedback $k_3$ affects both the undamped natural frequency and damping ratio of the closed loop system.

- The root locus is a circle with the origin at the point $n = -0.5 \text{rad/sec}$.

- Note that $k_3$ is varied towards infinity, the undamped natural frequency tends towards zero.
By selecting any combination value of $k_1, k_2, k_3$ it is possible to place the poles of the closed loop system at an arbitrary desired location in the S-plane
THANK YOU