FRAME CONVERSION
INERTIAL TO BODY FRAME

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A frame in which all Newton Law’s obeys.
Inertial Frame is also called non accelerating frame.
X-axis points north.
Y-axis points east.
Z-axis points towards down.
Inertial frame is also considered as NED Frame.
Note: Because the z-axis points down the altitude above the ground is negative.
Body frame is the coordinate system in which the frame is aligned with body of the sensor.

- X-axis point out of the nose
- Y-axis points out right side of the Fuselage
- Z-axis points out the bottom of the Fuselage

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INERTIAL FRAME TO VEHICLE 1 FRAME BY AN ANGLE ($\psi$)

\[ R_{v1}^{v1} (\psi) = \begin{pmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

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Vehicle Frame 1 to Vehicle Frame 2 by an Angle ($\theta$)

$$R_{v1}^{v2} (\theta) = \begin{pmatrix}
\cos(\theta) & 0 & -\sin(\theta) \\
0 & 1 & 0 \\
\sin(\theta) & 0 & \cos(\theta)
\end{pmatrix}$$

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Vehicle Frame 2 to Body Frame by an Angle \( (\phi) \)

\[
R^B_{v2} (\phi) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\phi) & \sin(\phi) \\
0 & -\sin(\phi) & \cos(\phi)
\end{bmatrix}
\]
CONVERSION FROM INERTIAL FRAME TO BODY FRAME

\[ R_I^B (\phi, \theta, \psi) = R_v^B (\phi) R_v^2 (\theta) R_I^v (\psi) \]

\[ R_I^B (\phi, \theta, \psi) = \begin{pmatrix}
C_\psi C_\theta & C_\theta S_\psi & -S_\theta \\
C_\psi S_\phi S_\theta - C_\phi S_\psi & C_\phi C_\psi + S_\phi S_\psi S_\theta & C_\theta S_\phi \\
S_\phi S_\psi + C_\phi C_\psi S_\theta & C_\phi S_\psi S_\theta - C_\psi S_\phi & C_\phi C_\theta
\end{pmatrix} \]

- The rotation matrix for moving opposite direction from body frame to the inertial frame.

\[ R_B^I (\phi, \theta, \psi) = R_I^v (-\psi) R_I^v (-\theta) R_v^2 (-\phi) \]

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The rategyro, accelerometer and magnetometer are aligned with the body frame of vehicle.

In order to get inertial frame data, the sensor outputs are converted from the body frame to the inertial frame.

This can be accomplished by performing the matrix multiplication

\[ R_B^I(\phi, \theta, \psi) = \begin{pmatrix} 
C_\psi C_\theta & C_\psi S_\phi S_\theta - C_\phi S_\psi & S_\phi S_\psi + C_\phi C_\psi S_\theta \\
C_\theta S_\psi & C_\phi C_\psi + S_\phi S_\psi S_\theta & C_\phi S_\psi S_\theta - C_\psi S_\phi \\
-S_\theta & C_\theta S_\phi & C_\phi C_\theta 
\end{pmatrix} \]

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• The resultant matrix for converting Body frame angular rates \((p, q, r)\) into Euler angular rate \((\dot{\phi}, \dot{\theta}, \dot{\psi})\) is

\[
\begin{bmatrix}
p \\ q \\ r
\end{bmatrix} = R^B_{\phi}(\dot{\phi}) \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + R^B_{\phi}(\dot{\phi})R^\phi_{\theta}(\dot{\theta}) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + R^B_{\phi}(\dot{\phi})R^\phi_{\theta}(\dot{\theta})R^\theta_\psi(\dot{\psi}) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}
\]

\[
R^\phi_\theta(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix}, \quad
R^\theta_\psi(\psi) = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix}
\]

\[
R^B_{\phi}(\phi) = \text{Identity Matrix}
\]
\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix} = \begin{bmatrix}
1 & 0 & -\sin(\theta) \\
0 & \cos(\phi) & \sin(\phi)\cos(\theta) \\
0 & -\sin(\phi) & \cos(\phi)\cos(\theta)
\end{bmatrix} \begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix}
\]
Inverting the relation gives relationship between body rate and Euler rate.

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = J \begin{bmatrix}
p \\
q \\
r
\end{bmatrix} = \begin{bmatrix}
1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\
0 & \cos(\phi) & -\sin(\phi) \\
0 & \sin(\phi) & \cos(\phi)
\end{bmatrix} \begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\]

\(J\) is the rotational matrix

\[
\begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} = \begin{bmatrix}
p + q\sin(\phi)\tan(\theta) + r\cos(\phi)\tan(\theta) \\
q\cos(\phi) - r\sin(\phi) \\
q\frac{\sin(\phi)}{\cos(\theta)} + r\frac{\cos(\phi)}{\cos(\theta)}
\end{bmatrix}
\]

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This operation explains mathematically why gimbal lock becomes a problem when using Euler Angles. To estimate yaw, pitch, and roll rates, gyro data must be converted to their proper coordinate frames using the matrix $J$. But notice that there is a division by in two places on the last row of the matrix.

When the pitch angle approaches +/- 90 degrees, the denominator goes to zero and the matrix elements diverge to infinity, causing the filter to fail.
Thank you