

LONGITUDINAL EQUATION OF MOTION

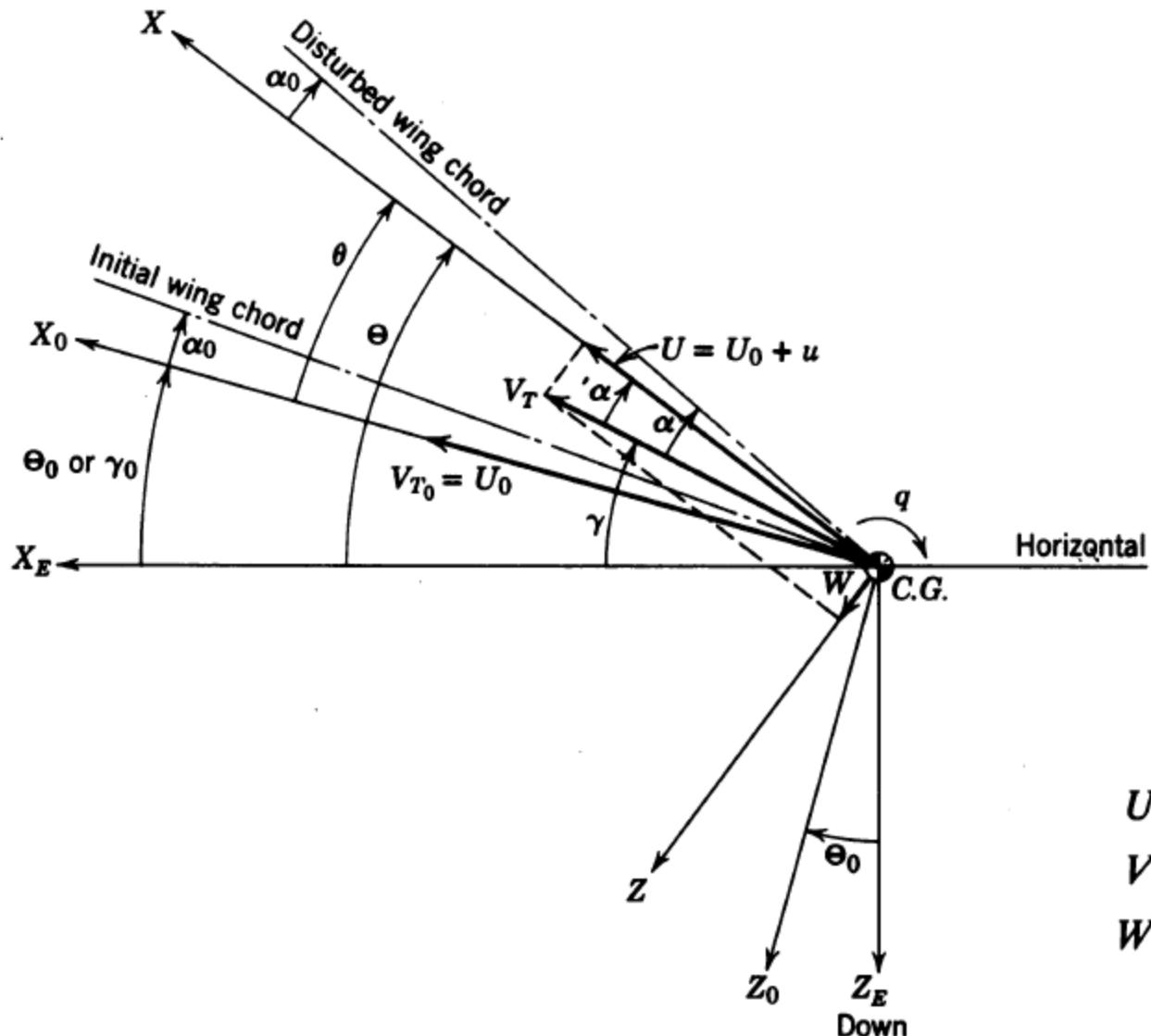


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Longitudinal Equation of motion

- The six aircraft equations of motion (EOM) can be decoupled into two sets of three equations. These are the three longitudinal EOM and the three lateral directional EOM.
- This is convenient in that it requires only three equations to be solved simultaneously for many flight conditions.
- For example, an aircraft in wings-level flight with no sideslip and a pitching motion can be analyzed using only the longitudinal EOM because the aircraft does not have any lateral-directional motion.



$$'\alpha \approx W / U$$

$$\alpha = \alpha_0 + '\alpha$$

$$\Theta = \Theta_0 + \theta$$

$$\gamma = \Theta - '\alpha$$

$$\text{or } \gamma = \theta - '\alpha \text{ for } \Theta_0 = 0$$

$U = U_0 + u$	$P = P_0 + p$
$V = V_0 + v$	$Q = Q_0 + q$
$W = W_0 + w$	$R = R_0 + r$

Equilibrium and disturbed aircraft stability axes

- The three longitudinal EOM consist of the x force, z force, and y moment equations, namely,

$$m(\dot{U} + QW - RV) = -mg \sin \Theta + (-D \cos A + L \sin A) + T \cos \phi_T$$

$$m(\dot{V} + RU - PW) = mg \sin \phi \cos \Theta + F_{Ay} + F_{Ty}$$

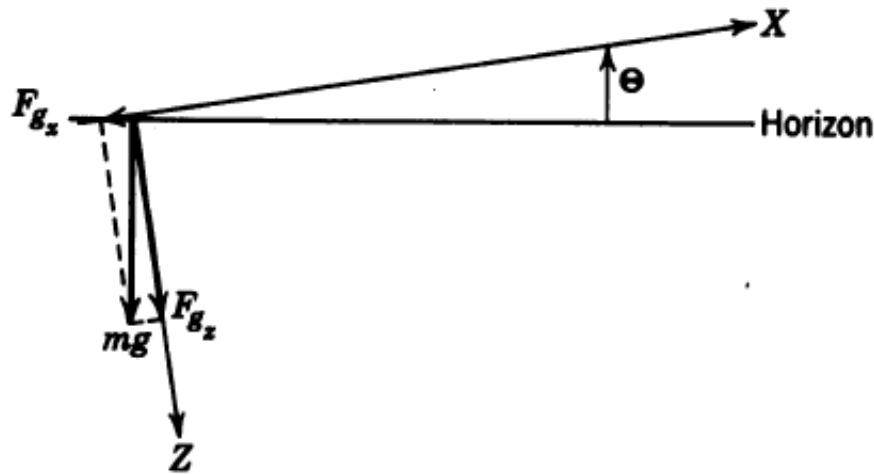
$$\underbrace{m(\dot{W} + PV - QU)}_{\text{Three force equation}} = mg \cos \phi \cos \Theta + (-D \sin A - L \cos A) - T \sin \phi_T$$

For longitudinal equation $P = R = V = 0$

$$\sum \Delta F_x = m(\dot{U} + WQ)$$

$$\dot{U} I_{yy} = M_A + M_T$$

$$\sum \Delta F_z = m(\dot{W} + QU)$$



Component of gravity resolved into aircraft axis

$$F_{gx} = -mg \sin(\Theta) \quad F_{gz} = mg \cos(\Theta)$$

$$\frac{\partial F_{gx}}{\partial \theta} = -mg \cos(\Theta) \quad \frac{\partial F_{gz}}{\partial \theta} = -mg \sin(\Theta)$$

The forces in the X direction are a function of $u, w, \dot{w}, \dot{\theta}, \dot{\Theta}$

Total differentiation of F_x

$$\sum \partial F_x = \frac{\partial F_x}{\partial u} dU + \frac{\partial F_x}{\partial w} dW + \frac{\partial F_x}{\partial \dot{w}} d\dot{W} + \frac{\partial F_x}{\partial \Theta} d\Theta + \frac{\partial F_x}{\partial \dot{\Theta}} d\dot{\Theta}$$

$$\sum \Delta F_x = \frac{\partial F_x}{\partial u} \Delta U + \frac{\partial F_x}{\partial w} \Delta W + \frac{\partial F_x}{\partial \dot{w}} \Delta \dot{W} + \frac{\partial F_x}{\partial \Theta} \Delta \Theta + \frac{\partial F_x}{\partial \dot{\Theta}} \Delta \dot{\Theta}$$

$\Delta u = u$ (because of small disturbance)

$$\sum \Delta F_x = \frac{\partial F_x}{\partial u} u + \frac{\partial F_x}{\partial w} w + \frac{\partial F_x}{\partial \dot{w}} \dot{w} + \frac{\partial F_x}{\partial \Theta} \Theta + \frac{\partial F_x}{\partial \dot{\Theta}} \dot{\Theta}$$

$u, w, \text{ etc}$ are the changes in the parameters and as the perturbations are small.

$$u = \Delta U$$

$$\therefore \Theta = \theta$$

$$\sum \Delta F_x = \frac{\partial F_x}{\partial u} u + \frac{\partial F_x}{\partial w} w + \frac{\partial F_x}{\partial \dot{w}} \dot{w} + \frac{\partial F_x}{\partial \theta} \theta + \frac{\partial F_x}{\partial \dot{\theta}} \dot{\theta}$$

multiply and divide the 1st three terms by U_0

$$\sum \Delta F_x = U_0 \frac{\partial F_x}{\partial u} \frac{u}{U_0} + U_0 \frac{\partial F_x}{\partial w} \frac{w}{U_0} + U_0 \frac{\partial F_x}{\partial \dot{w}} \frac{\dot{w}}{U_0} + \frac{\partial F_x}{\partial \theta} \theta + \frac{\partial F_x}{\partial \dot{\theta}} \dot{\theta}$$

$$U_0 = U$$

$$\sum \Delta F_x = U \frac{\partial F_x}{\partial u} \frac{u}{U} + U \frac{\partial F_x}{\partial w} \frac{w}{U} + U \frac{\partial F_x}{\partial \dot{w}} \frac{\dot{w}}{U} + \frac{\partial F_x}{\partial \theta} \theta + \frac{\partial F_x}{\partial \dot{\theta}} \dot{\theta}$$

$$\frac{u}{U} = u' \quad \frac{w}{U} = \alpha' \quad \frac{\dot{w}}{U} = \dot{\alpha}$$

$$\sum \Delta F_x = U \frac{\partial F_x}{\partial u} u' + U \frac{\partial F_x}{\partial w} \dot{\alpha} + U \frac{\partial F_x}{\partial \dot{w}} \ddot{\alpha} + \frac{\partial F_x}{\partial \theta} \theta + \frac{\partial F_x}{\partial \dot{\theta}} \dot{\theta}$$

$$U \frac{\partial F_x}{\partial w} = \frac{\partial F_x}{\partial w/U} = \frac{\partial F_x}{\partial \dot{\alpha}} = \frac{\partial F_x}{\partial \alpha} \quad \text{as} \quad \frac{\partial \dot{\alpha}}{\partial \alpha} \equiv 1$$

$$\sum \Delta F_x = m\dot{u} = m\dot{u} \left(\frac{U}{U} \right) \text{ same U on denominator and neumarotor}$$

$$\sum \Delta F_x = m\dot{u} = mU \left(\frac{\dot{u}}{U} \right) \quad \frac{\dot{u}}{U} = \dot{u}$$

$$\sum \Delta F_x = m\dot{u} = mU \dot{u}$$

$$mU \dot{u} = U \frac{\partial F_x}{\partial u} u' + U_0 \frac{\partial F_x}{\partial \alpha} \alpha' + U \frac{\partial F_x}{\partial \dot{\alpha}} \dot{\alpha}' + \frac{\partial F_x}{\partial \theta} \theta + \frac{\partial F_x}{\partial \dot{\theta}} \Delta \dot{\theta}$$

$$mU \ddot{u} - U \frac{\partial F_x}{\partial u} \dot{u} - U_0 \frac{\partial F_x}{\partial \alpha} \dot{\alpha} - U \frac{\partial F_x}{\partial \dot{\alpha}} \ddot{\alpha} - \frac{\partial F_x}{\partial \theta} \dot{\theta} - \frac{\partial F_x}{\partial \dot{\theta}} \ddot{\theta} = F_{xa}$$

F_{xa} = applied aerodynamics force

Divided by Sq in the above equation

$$\frac{mU}{Sq} \ddot{u} - \frac{U}{Sq} \frac{\partial F_x}{\partial u} \dot{u} - \frac{1}{Sq} \frac{\partial F_x}{\partial \alpha} \dot{\alpha} - \frac{1}{Sq} \frac{\partial F_x}{\partial \dot{\alpha}} \ddot{\alpha} - \frac{1}{Sq} \frac{\partial F_x}{\partial \theta} \dot{\theta} - \frac{mU}{Sq} \frac{\partial F_x}{\partial \dot{\theta}} \ddot{\theta} = \frac{F_{xa}}{Sq}$$

s-wing Area

$$q = \frac{1}{2} \rho v^2 (\text{Dynamic pressure})$$

multiply and divide the $\frac{\partial F_x}{\partial \dot{\alpha}}, \frac{\partial F_x}{\partial \theta}, \frac{\partial F_x}{\partial \dot{\theta}}$ *terms by* $\frac{c}{2U}$

c-mean aerodynamic chord

$$\frac{mU}{Sq} \ddot{u} - \frac{U}{Sq} \frac{\partial F_x}{\partial u} \dot{u} - \frac{1}{Sq} \frac{\partial F_x}{\partial \alpha} \dot{\alpha} - \frac{c}{2U} \left(\frac{1}{sq} \right) \frac{2U}{c} \frac{\partial F_x}{\partial \dot{\alpha}} \dot{\alpha} + \frac{mg}{sq} (\cos \Theta) \theta - \frac{c}{2U} \left(\frac{1}{sq} \right) \frac{2U}{c} \frac{\partial F_x}{\partial \dot{\theta}} \dot{\theta} = \frac{F_{xa}}{sq} = C_{Fxa}$$

$$\frac{mU}{Sq} \ddot{u} - C_{xu} \dot{u} - C_{x\alpha} \dot{\alpha} - \frac{c}{2U} C_{x\dot{\alpha}} \dot{\alpha} + \frac{mg}{sq} (\cos \Theta) \theta - \frac{c}{2U} C_{xq} \dot{\theta} = \frac{F_{xa}}{sq} = C_{Fxa}$$

$$C_{xu} = \frac{U}{Sq} \frac{\partial F_x}{\partial u}, C_{x\alpha} = \frac{1}{Sq} \frac{\partial F_x}{\partial \alpha}, C_{x\dot{\alpha}} = \left(\frac{1}{sq} \right) \frac{2U}{c} \frac{\partial F_x}{\partial \dot{\alpha}}, C_{xq} \dot{\theta} = \left(\frac{1}{sq} \right) \frac{2U}{c} \frac{\partial F_x}{\partial \dot{\theta}}$$

$$C_w = \frac{mg}{sq}$$

$$\frac{mU}{Sq} \ddot{u} - C_{xu} \dot{u} - C_{x\alpha} \dot{\alpha} - \frac{c}{2U} C_{x\dot{\alpha}} \dot{\alpha} - C_w (\cos \Theta) \theta - \frac{c}{2U} C_{xq} \dot{\theta} = \frac{F_{xa}}{sq} = C_{Fxa}$$

The forces in the Z direction are a function of $u, w, \dot{w}, \theta, \dot{\theta}$

$$\sum \Delta F_z = \frac{\partial F_z}{\partial u} u + \frac{\partial F_z}{\partial w} w + \frac{\partial F_z}{\partial \dot{w}} \dot{w} + \frac{\partial F_z}{\partial \theta} \theta + \frac{\partial F_z}{\partial \dot{\theta}} \dot{\theta}$$

multiply and divide the 1st three terms by U

$$\sum \Delta F_z = U \frac{\partial F_z}{\partial u} \frac{u}{U} + U \frac{\partial F_z}{\partial w} \frac{w}{U} + U \frac{\partial F_z}{\partial \dot{w}} \frac{\dot{w}}{U} + \frac{\partial F_z}{\partial \theta} \theta + \frac{\partial F_z}{\partial \dot{\theta}} \dot{\theta}$$

$$\sum \Delta F_z = U \frac{\partial F_z}{\partial u} \dot{u} + U \frac{\partial F_z}{\partial w} \dot{\alpha} + U \frac{\partial F_z}{\partial \dot{w}} \ddot{\alpha} + \frac{\partial F_z}{\partial \theta} \theta + \frac{\partial F_z}{\partial \dot{\theta}} \dot{\theta}$$

Note: $U \frac{\partial F_z}{\partial w} = \frac{\partial F_z}{\partial w/U} = \frac{\partial F_z}{\partial \alpha} = \frac{\partial F_z}{\partial \dot{\alpha}}$ as $\frac{\partial \alpha}{\partial \dot{\alpha}} \equiv 1$

$$\frac{u}{U} = \dot{u} \quad \frac{w}{U} = \dot{\alpha} \quad \frac{\dot{w}}{U} = \ddot{\alpha}$$

$$\sum \Delta F_z = m(\dot{W} + PV - QU)$$

$$P = V = 0$$

$$\sum \Delta F_z = m(\dot{W} - QU)$$

$$= m(\dot{W} - \dot{\theta}U) \quad (Q = \dot{\theta})$$

$$= m\left(\frac{\dot{W}}{U}U - \dot{\theta}U\right)$$

$$\frac{u}{U} = \dot{u} \quad \frac{w}{U} = \dot{\alpha} \quad \frac{\dot{w}}{U} = \dot{\dot{\alpha}}$$

$$= m\dot{\alpha}U - m\dot{\theta}U$$

$$m\dot{\alpha}U - m\dot{\theta}U = U \frac{\partial F_z}{\partial u} \dot{u} + \frac{\partial F_z}{\partial \alpha} \dot{\alpha} + U \frac{\partial F_z}{\partial \dot{\alpha}} \ddot{\alpha} + \frac{\partial F_z}{\partial \theta} \dot{\theta} + \frac{\partial F_z}{\partial \dot{\theta}} \ddot{\theta}$$

Divided by Sq

$$\frac{m\dot{\alpha}U}{Sq} - \frac{m\dot{\theta}U}{Sq} = U \frac{\partial F_z}{\partial u} \dot{u} + \frac{\partial F_z}{\partial \alpha} \dot{\alpha} + U \frac{\partial F_z}{\partial \dot{\alpha}} \ddot{\alpha} + \frac{\partial F_z}{\partial \theta} \dot{\theta} + \frac{\partial F_z}{\partial \dot{\theta}} \ddot{\theta}$$

$$\frac{m\dot{\alpha}U}{Sq} - \frac{m\dot{\theta}U}{Sq} = \frac{U}{Sq} \frac{\partial F_z}{\partial u} \dot{u} + \frac{1}{Sq} U \frac{\partial F_z}{\partial \alpha} \dot{\alpha} + U \frac{1}{Sq} \frac{\partial F_z}{\partial \dot{\alpha}} \dot{\alpha} + \frac{1}{Sq} \frac{\partial F_z}{\partial \theta} \theta + \frac{1}{Sq} \frac{\partial F_z}{\partial \dot{\theta}} \dot{\theta}$$

$$-\frac{U}{Sq} \frac{\partial F_z}{\partial u} \dot{u} + \frac{m\dot{\alpha}U}{Sq} - \frac{1}{Sq} \frac{\partial F_z}{\partial \dot{\alpha}} \dot{\alpha} - \frac{1}{Sq} \frac{\partial F_z}{\partial \alpha} \alpha - \frac{m\dot{\theta}U}{Sq} - \frac{1}{Sq} \frac{\partial F_z}{\partial \dot{\theta}} \dot{\theta} - \frac{1}{Sq} \frac{\partial F_z}{\partial \theta} \theta = C_{Fxa}$$

$$-\frac{U}{Sq} \frac{\partial F_z}{\partial u} \dot{u} + \left(\frac{mU}{Sq} - \frac{1}{Sq} \frac{\partial F_z}{\partial \dot{\alpha}} \right) \dot{\alpha} - \frac{1}{Sq} \frac{\partial F_z}{\partial \alpha} \alpha + \left(-\frac{mU}{Sq} - \frac{1}{Sq} \frac{\partial F_z}{\partial \dot{\theta}} \right) \dot{\theta} - \frac{1}{Sq} \frac{\partial F_z}{\partial \theta} \theta = \frac{Fza}{Sq} = C_{Fza}$$

$$-C_{zu} \dot{u} + \left(\frac{mU}{Sq} - \frac{c}{2U} C_{z\dot{\alpha}} \right) \dot{\alpha} - C_{z\alpha} \alpha + \left(-\frac{mU}{Sq} - \frac{c}{2U} C_{zq} \right) \dot{\theta} - Cw (\sin \Theta) \theta = \frac{Fza}{Sq} = C_{Fza}$$

$$C_{zu} = \frac{U}{Sq} \frac{\partial F_z}{\partial u}, C_{z\dot{\alpha}} = \frac{1}{Sq} \frac{2U}{c} \frac{\partial F_z}{\partial \dot{\alpha}}, C_{z\alpha} = \frac{1}{Sq} \frac{\partial F_z}{\partial \alpha}, C_{zq} = \frac{1}{Sq} \frac{2U}{c} \frac{\partial F_z}{\partial \dot{\theta}}, Cw = \frac{1}{Sq} \frac{\partial F_z}{\partial \theta}$$

similarly for moment about yaxis

$$\sum \Delta M = \frac{\partial M}{\partial u} u + \frac{\partial M}{\partial w} w + \frac{\partial M}{\partial \dot{w}} \dot{w} + \frac{\partial M}{\partial \theta} \theta + \frac{\partial M}{\partial \dot{\theta}} \dot{\theta}$$

$$\frac{\partial M}{\partial \theta} = 0 \text{ (there is no change in } M \text{ due to change in } \theta)$$

$$\sum \Delta M = U \frac{\partial M}{\partial u} u + \frac{\partial M}{\partial \alpha} \alpha + \frac{\partial M}{\partial \dot{\alpha}} \dot{\alpha} + \frac{\partial M}{\partial \dot{\theta}} \dot{\theta}$$

we know that $\sum \Delta M = I_y \ddot{\theta}$

$$I_y \ddot{\theta} = U \frac{\partial M}{\partial u} u + \frac{\partial M}{\partial \alpha} \alpha + \frac{\partial M}{\partial \dot{\alpha}} \dot{\alpha} + \frac{\partial M}{\partial \dot{\theta}} \dot{\theta}$$

$$\frac{Iy\ddot{\theta}}{Sqc} = \frac{U}{Sqc} \frac{\partial M}{\partial u} \dot{u} + \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\alpha}} \dot{\alpha} + \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\alpha}} \dot{\alpha} + \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\theta}} \dot{\theta}$$

After dividing by Sqc it becomes

$$\frac{U}{Sqc} \frac{\partial M}{\partial u} \dot{u} + \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\alpha}} \dot{\alpha} + \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\alpha}} \dot{\alpha} - \frac{Iy\ddot{\theta}}{Sqc} + \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\theta}} \dot{\theta} = -C_{m\alpha}$$

$$\frac{U}{Sqc} \frac{\partial M}{\partial u} \dot{u} + \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\alpha}} \dot{\alpha} + \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\alpha}} \dot{\alpha} - \frac{Iy\ddot{\theta}}{Sqc} + \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\theta}} \dot{\theta} = -C_{m\alpha}$$

$$-\frac{U}{Sqc} \frac{\partial M}{\partial u} \dot{u} - \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\alpha}} \dot{\alpha} - \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\alpha}} \dot{\alpha} + \frac{Iy\ddot{\theta}}{Sqc} - \frac{1}{Sqc} \frac{\partial M}{\partial \dot{\theta}} \dot{\theta} = C_{m\alpha}$$

$$-C_{mu} \dot{u} - \frac{c}{2U} C_{m\dot{\alpha}} \dot{\alpha} - C_{m\alpha} \dot{\alpha} + \frac{Iy\ddot{\theta}}{Sqc} - \frac{c}{2U} C_{mq} \dot{\theta} = C_{m\alpha}$$

$$\left(\frac{mU}{Sq} \dot{u} - C_{xu} u \right) + \left(-\frac{c}{2U} C_{x\dot{\alpha}} \dot{\alpha} - C_{x\alpha} \dot{\alpha} \right) + \left(-\frac{c}{2U} C_{xq} \dot{\theta} - C_w (\cos \Theta) \theta \right) = \frac{F_{xa}}{sq} = C_{Fxa}$$

$$-C_{zu} u + \left[\left(\frac{mU}{Sq} - \frac{c}{2U} C_{z\dot{\alpha}} \right) \dot{\alpha} - C_{z\alpha} \dot{\alpha} \right] + \left[\left(-\frac{mU}{Sq} - \frac{c}{2U} C_{zq} \right) \dot{\theta} - C_w (\sin \Theta) \theta \right] = \frac{F_{za}}{sq} = C_{Fza}$$

$$(-C_{mu} u) - \left(\frac{c}{2U} C_{m\dot{\alpha}} \dot{\alpha} - C_{m\alpha} \dot{\alpha} \right) + \left(\frac{Iy \ddot{\theta}}{Sqc} - \frac{c}{2U} C_{mq} \dot{\theta} \right) = C_{m\alpha}$$

TABLE 1-1 Definitions and Equations for Longitudinal Stability Derivatives

Symbol	Definition	Origin	Equation	Typical Values
C_{x_u}	$\frac{U}{Sg} \frac{\partial F_x}{\partial u}$	Variation of drag and thrust with u	$-2C_D - U \frac{\partial C_D}{\partial u}$	-0.05
C_{x_α}	$\frac{1}{Sg} \frac{\partial F_x}{\partial \alpha}$	Lift and drag variations along the X axis	$C_L - \frac{\partial C_D}{\partial \alpha}$	+0.1
C_w	—	Gravity	$-\frac{mg}{Sg}$	
C_{x_d}	$\frac{1}{Sg} \left(\frac{2U}{c} \right) \frac{\partial F_x}{\partial \dot{\alpha}}$	Downwash lag on drag	Neglect	
C_{x_θ}	$\frac{1}{Sg} \left(\frac{2U}{c} \right) \frac{\partial F_x}{\partial \dot{\theta}}$	Effect of pitch rate on drag	Neglect	
C_{z_u}	$\frac{U}{Sg} \frac{\partial F_z}{\partial u}$	Variation of normal force with u	$-2C_L - U \frac{\partial C_L}{\partial u}$	-0.5
C_{z_α}	$\frac{1}{Sg} \frac{\partial F_z}{\partial \alpha}$	Slope of the normal force curve	$-C_D - \frac{\partial C_L}{\partial \alpha} \approx -\frac{\partial C_L}{\partial \alpha}$	-4
C_w	—	Gravity	$-\frac{mg}{Sg}$	
C_{z_d}	$\frac{1}{Sg} \left(\frac{2U}{c} \right) \frac{\partial F_z}{\partial \dot{\alpha}}$	Downwash lag on lift of tail	$2 \left(\frac{\partial C_m}{\partial i_t} \right) \left(\frac{d\epsilon}{d\alpha} \right)$	-1
C_{z_θ}	$\frac{1}{Sg} \left(\frac{2U}{c} \right) \frac{\partial F_z}{\partial \dot{\theta}}$	Effect of pitch rate on lift	$2K \left(\frac{\partial C_m}{\partial i_t} \right)$	-2
C_{m_u}	$\frac{U}{Sqc} \frac{\partial \mathcal{M}}{\partial u}$	Effects of thrust, slipstream, and flexibility	No simple relation; usually neglected for jets	
C_{m_α}	$\frac{1}{Sqc} \frac{\partial \mathcal{M}}{\partial \alpha}$	Static longitudinal stability	$(SM) \left(\frac{dC_L}{d\alpha} \right)_s$	-0.3
C_{m_d}	$\frac{1}{Sqc} \left(\frac{2U}{c} \right) \frac{\partial \mathcal{M}}{\partial \dot{\alpha}}$	Downwash lag on moment	$2 \left(\frac{\partial C_m}{\partial i_t} \right) \frac{d\epsilon}{d\alpha} \frac{l_t}{c}$	-3
C_{m_θ}	$\frac{1}{Sqc} \left(\frac{2U}{c} \right) \frac{\partial \mathcal{M}}{\partial \dot{\theta}}$	Damping in pitch	$2K \left(\frac{\partial C_m}{\partial i_t} \right) \frac{l_t}{c}$	-8

- The aircraft is flying in straight and level flight at 40,000 ft with a velocity of 600 ft/sec (355 knots), and the compressibility effects will be neglected. For this aircraft the values are as follows

$$\Theta = 0$$

$$Mach = 0.62$$

$$m = 5800 \text{ slugs}$$

$$U = 600 \text{ ft/sec}$$

$$S = 2400 \text{ sq ft}$$

$$I_y = 2.62 \times 10^6 \text{ slug ft}^2$$

$$C_{xu} = -2C_D = -0.088$$

$$C_{x\alpha} = -\frac{\partial C_D}{\partial \alpha} + C_L = 0.392$$

$$C_w = -\frac{mg}{Sq} = -C_L = -0.74$$

$$\frac{l_t}{c} = 2.89$$

$$c = 20.2 \text{ ft}$$

$$C_{zu} = -2C_L = -1.48$$

$$C_{z\dot{\alpha}} = \left(\frac{dC_m}{di_t} \right)_{\delta,\alpha}^t \left(\frac{\partial C_L}{\partial \alpha} \right)(2) = (-1.54)(0.367)(2) = -1.13$$

$$C_{z\alpha} = -\left(\frac{\partial C_L}{\partial \alpha} \right) - C_D = -4.42 - 0.04 = -4.46$$

$$C_{zq} = 2K \left(\frac{dC_m}{di_t} \right)_{\delta,\alpha}^t = 2.56(-1.54) = -3.94$$

$$C_{m\dot{\alpha}} = 2 \left(\frac{dC_m}{di} \right)_{\delta,\alpha}^t \left(\frac{\partial \in}{\partial \alpha} \right) \frac{lt}{c} = (-1.54)(0.367)(2)(2.89) = -3.27$$

$$C_{m\alpha} = (SM)_{\delta} \left(\frac{\partial C_L}{\partial \alpha} \right)_{\delta}^a = (-0.14)(4.42) = -0.619$$

$$C_{mq} = 2K \left(\frac{dC_m}{di} \right)_{\delta,\alpha}^t \frac{lt}{c} = (2.56)(-1.54)(2.89) = -11.4$$

$$q = \frac{\rho}{2} U^2 = \frac{(0.000585)(600)^2}{2} = 105.1 \text{ lb/sq ft}$$

$$\frac{mU}{Sq} = \frac{(5800)(600)}{(2400)(105.1)} = 13.78 \text{ sec}$$

$$\frac{c}{2U} C_{z\dot{\alpha}} = \frac{(20.2)(-1.13)}{(2)(600)} = -0.019 \text{ sec}$$

$$\frac{c}{2U} C_{zq} = (0.0168)(-3.94) = -0.066 \text{ sec}$$

$$\frac{c}{2U} C_{m\dot{\alpha}} = (0.0168)(-3.27) = -0.0552 \text{ sec}$$

$$\frac{c}{2U} C_{mq} = (0.0168)(-11.4) = -0.192 \text{ sec}$$

$$\frac{I_y}{Sq c} = \frac{(2.62 \times 10^6)}{(2400)(105.1)(20.2)} = 0.514 \text{ sec}^2$$

$$\left(\frac{mU}{Sq} \dot{u} - C_{xu} u \right) + \left(-\frac{c}{2U} C_{x\dot{\alpha}} \dot{\alpha} - C_{x\alpha} \dot{\alpha} \right) + \left(-\frac{c}{2U} C_{xq} \dot{\theta} - C_w (\cos \Theta) \theta \right) = \frac{F_{xa}}{sq} = C_{Fxa}$$

$$(13.78s + 0.088)u(s) - 0.392\alpha(s) + 0.74\theta(s) = 0$$

$$-C_{zu} \dot{u} + \left[\left(\frac{mU}{Sq} - \frac{c}{2U} C_{z\dot{\alpha}} \right) \dot{\alpha} - C_{z\alpha} \dot{\alpha} \right] + \left[\left(-\frac{mU}{Sq} - \frac{c}{2U} C_{zq} \right) \dot{\theta} - C_w (\sin \Theta) \theta \right] = \frac{F_{za}}{sq} = C_{Fza}$$

$$(1.48)u(s) + (13.78s + 4.46)\alpha(s) - 13.78s\theta(s) = 0$$

$$(-C_{mu} \dot{u}) - \left(\frac{c}{2U} C_{m\dot{\alpha}} \dot{\alpha} - C_{m\alpha} \dot{\alpha} \right) + \left(\frac{Iy \ddot{\theta}}{Sqc} - \frac{c}{2U} C_{mq} \dot{\theta} \right) = C_{m\alpha}$$

$$0 + (0.0552s + 0.619)\alpha(s) + (0.514s^2 + 0.192s)\theta(s) = 0$$

$$\begin{bmatrix} 13.78s + 0.088 & -0.392 & 0.74\theta \\ 1.48 & 13.78s + 4.46 & -13.78s \\ 0 & 0.0552s + 0.619 & 0.514s^2 + 0.192s \end{bmatrix} = 0$$

$$97.5s^4 + 79s^3 + 128.9s^2 + 0.998s + 0.677 = 0$$

Dividing by the s^4 coefficient

$$s^4 + 0.811s^3 + 1.32s^2 + 0.0102s + 0.00695 = 0$$

$$(s^2 + 0.00466s + 0.0053)(s^2 + 0.806s + 1.311) = 0$$

$$(s^2 + 2\delta\omega_p s + \omega_p^2) = (s^2 + 0.00466s + 0.0053) \quad (s^2 + 2\delta\omega_s s + \omega_s^2) = (s^2 + 0.806s + 1.311)$$

$$\omega_p = 0.073 \text{ rad/sec}$$

$$\delta p = 0.032$$

$$\omega_{ns}^2 = 1.145 \text{ rad/sec}$$

$$\delta s = 0.352$$

Thank you