FRAME CONVERSION
INERTIAL TO BODY FRAME

Prepared
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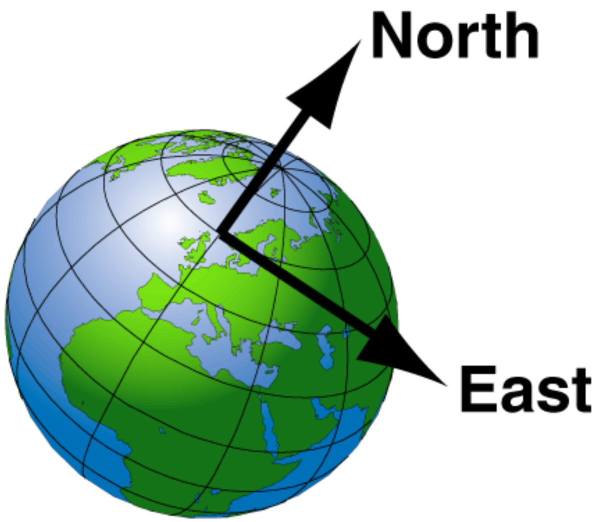
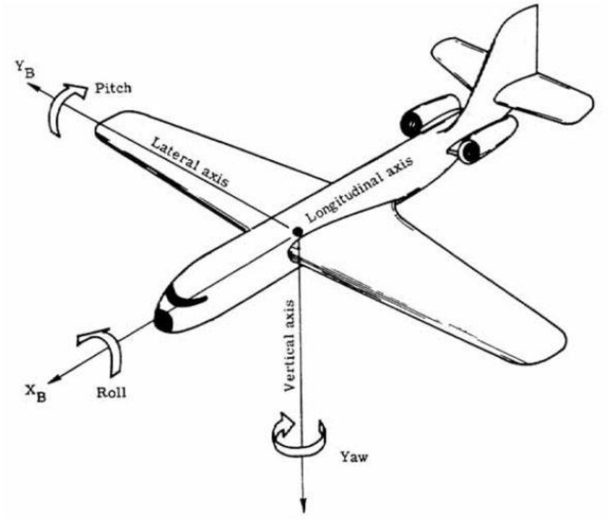
INERTIAL FRAME

- A frame in which all Newton Law's obeys.
- Inertial Frame is also called non accelerating frame.
- X-axis points north.
- Y-axis points east.
- Z-axis points towards down.
- Inertial frame is also consider as NED Frame.
- Note: Because the z-axis points down the altitude above the ground is negative.



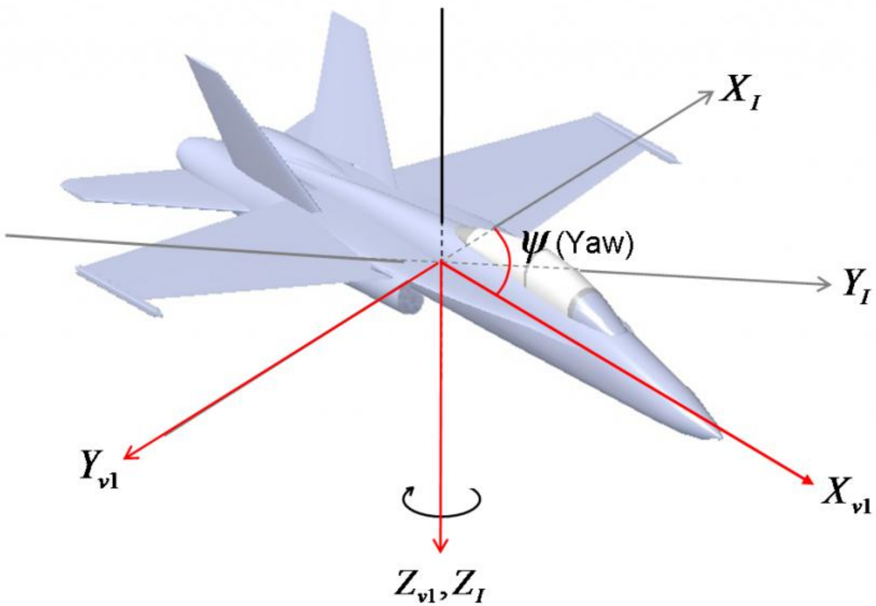
BODY FRAME

- Body frame is the coordinate system in which the frame is aligned with body of the sensor.
- X-axis point out of the nose
- Y-axis points out right side of the Fuselage
- Z-axis points out the bottom of the Fuselage



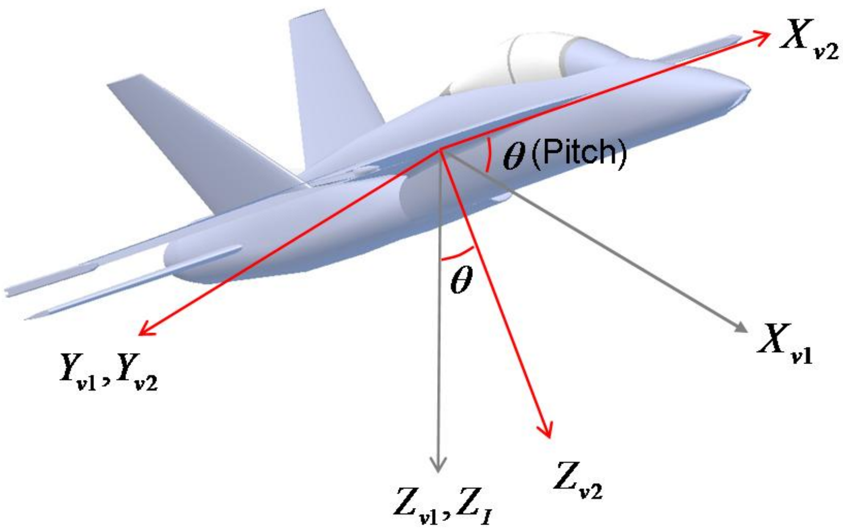
Conversion from Inertial Frame to Body Frame

Inertial Frame to Vehicle 1 Frame by an Angle (ψ)



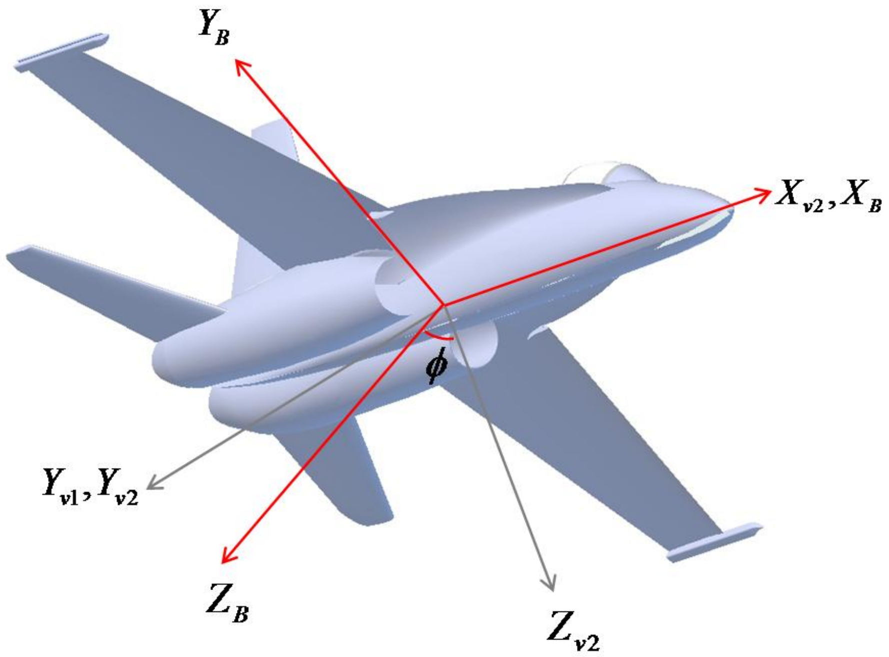
$$R_I^{v1}(\psi) = \begin{pmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Vehicle Frame 1 to Vehicle Frame 2 by an Angle (θ)



$$R_{v1}^{v2}(\theta) = \begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$

Vehicle Frame 2 to Body Frame by an Angle (ϕ)



$$R_{v2}^{B}(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix}$$

CONVERSION FROM INERTIAL FRAME TO BODY FRAME

$$R_I^B(\phi,\theta,\psi) = R_{v2}^B(\phi)\, R_{v1}^{v2}(\theta)\, R_I^{v1}(\psi)$$

$$R_I^B(\phi,\theta,\psi) = \begin{pmatrix} C_\psi C_\theta & C_\theta S_\psi & -S_\theta \\ C_\psi S_\phi S_\theta - C_\phi S_\psi & C_\phi C_\psi + S_\phi S_\psi S_\theta & C_\theta S_\phi \\ S_\phi S_\psi + C_\phi C_\psi S_\theta & C_\phi S_\psi S_\theta - C_\psi S_\phi & C_\phi C_\theta \end{pmatrix}$$

- The rotation matrix for moving opposite direction from body frame to the inertial frame.

$$R_B^I(\phi,\theta,\psi) = R_I^{v1}(-\psi)\, R_{v1}^{v2}(-\theta)\, R_{v2}^B(-\phi)$$

$$R_B^I(\phi,\theta,\psi)=\begin{pmatrix} C_\psi C_\theta & C_\psi S_\phi S_\theta - C_\phi S_\psi & S_\phi S_\psi + C_\phi C_\psi S_\theta \\ C_\theta S_\psi & C_\phi C_\psi + S_\phi S_\psi S_\theta & C_\phi S_\psi S_\theta - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\phi C_\theta \end{pmatrix}$$

- The rategyro,accelerometer and magnetometer are aligned with the body frame of vehicle.
- In order to get inertial frame data ,the sensor outputs are converted from the body frame to the inertial frame.
- This can be accomplished by performing the matrix multiplication $R_B^I(\phi,\theta,\psi)$.

- The resultant matrix for converting Body frame angular rates (p,q,r) into Euler angular rate $(\dot{\phi}, \dot{\theta}, \dot{\psi})$ is

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = R_{\phi}^{B}(\phi)\begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + R_{\phi}^{B}(\phi)R_{\theta}^{\phi}(\theta)\begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + R_{\phi}^{B}(\phi)R_{\theta}^{\phi}(\theta)R_{I}^{\theta}(\psi)\begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$$

$$R_{\theta}^{\phi}(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix} \qquad R_{I}^{\theta}(\psi) = \begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$

$$R_{\phi}^{B}(\phi) = \text{Identity Matrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{pmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\phi) & \sin(\phi)\cos(\theta) \\ 0 & -\sin(\phi) & \cos(\phi)\cos(\theta) \end{pmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

Inverting the relation gives relationship between body rate and Euler rate.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = J \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{pmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \dfrac{\sin(\phi)}{\cos(\theta)} & \dfrac{\cos(\phi)}{\cos(\theta)} \end{pmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

J is the rotational matrix

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{pmatrix} p + \mathrm{q}\sin(\phi)\tan(\theta) + r\cos(\phi)\tan(\theta) \\ q\cos(\phi) - r\sin(\phi) \\ q\dfrac{\sin(\phi)}{\cos(\theta)} + r\dfrac{\cos(\phi)}{\cos(\theta)} \end{pmatrix}$$

- This operation explains mathematically why gimbal lock becomes a problem when using Euler Angles. To estimate yaw, pitch, and roll rates, gyro data must be converted to their proper coordinate frames using the matrix *J*. But notice that there is a division by in two places on the last row of the matrix.

- When the pitch angle approaches +/- 90 degrees, the denominator goes to zero and the matrix elements diverge to infinity, causing the filter to fail.

Thank you