

LATERAL EQUATION OF MOTION



Prepared
by

A.Kaviyarasu
Assistant Professor
Department of Aerospace Engineering
Madras Institute Of Technology
Chromepet, Chennai

$$m(\dot{U} + QW - RV) = F_{Gx} + F_{Ax} + F_{Tx}$$

$$m(\dot{V} + RU - PW) = F_{Gy} + F_{Ay} + F_{Ty} \leftarrow \text{one force equation}$$

$$m(\dot{W} + PV - QU) = F_{Gz} + F_{Az} + F_{Tz}$$

$$\dot{P}I_{xx} + QR(I_{zz} - I_{yy}) - (\dot{R} + PQ)I_{xz} = L \quad \leftarrow \text{Moment equation}$$

$$\dot{Q}I_{yy} - PR(I_{zz} - I_{xx}) + (P^2 - R^2)I_{xz} = M$$

$$\dot{R}I_{zz} + PQ(I_{yy} - I_{xx}) + (QP - \dot{P})I_{xz} = N \quad \leftarrow \text{Moment equation}$$

In the above equation Q is assumed to be zero. Since it does not have any effect on lateral axis

As the aircraft initially is in unaccelerated flight $P_0 = R_0 = 0$ then $P = p$ and $R = r$

$$\sum \Delta F_y = m(\dot{v} + U_0 r + u r) \quad (\dot{V} = \dot{v}) \text{ at equilibrium no slideslip } V_0 = W = 0$$

$$\sum \Delta L = \dot{p} I_x - \dot{r} J_{xz} \quad (\dot{P} = \dot{p}, \dot{R} = \dot{r}, I_{xx} = I_x \text{ and } I_{xz} = J_{xz})$$

$$\sum \Delta N = \dot{r} I_z - \dot{p} J_{xz} \quad (\dot{R} = \dot{r}, I_{zz} = I_z \text{ and } I_{xz} = J_{xz})$$

$$\sum \Delta F_y = m(\dot{v} + U_0 r)$$

$$\sum \Delta L = \dot{p}I_x - \dot{r}J_{xz}$$

$$\sum \Delta N = \dot{r}I_z - \dot{p}J_{xz}$$

However, since the perturbations are assumed small, the products of the perturbations can be neglected. The above equation then reduces to

$$\sum \Delta F_y = mU_0 \left(\frac{\dot{v}}{U_0} + r \right)$$

$$\dot{\beta} = \frac{\dot{v}}{U_0}$$

$$\sum \Delta F_y = mU(\dot{\beta} + r) = mU\dot{\beta} + mUr$$

$$U_0 = U$$

$$\sum \Delta F_y = mU\dot{\beta} + mU\dot{\psi}$$

$$\sum \Delta L = \ddot{\phi}I_x - \ddot{\psi}J_{xz}$$

$$\sum \Delta N = \ddot{\psi}I_z - \ddot{\phi}J_{xz}$$

$$\ddot{\psi} = \dot{r}$$

$$\ddot{\phi} = \dot{p}$$

FORCE EQUATION ALONG Y AXIS

The forces in the y direction are function of $\beta, \psi, \phi, \dot{\phi}$ and $\dot{\psi}$

$$\sum \partial F_y = \frac{\partial F_y}{\partial \beta} d\beta + \frac{\partial F_y}{\partial \psi} d\psi + \frac{\partial F_y}{\partial \phi} d\phi + \frac{\partial F_y}{\partial \dot{\phi}} \partial \dot{\phi} + \frac{\partial F_y}{\partial \dot{\psi}} \partial \dot{\psi}$$

$$\sum \partial F_y = \frac{\partial F_y}{\partial \beta} \Delta \beta + \frac{\partial F_y}{\partial \psi} \Delta \psi + \frac{\partial F_y}{\partial \phi} \Delta \phi + \frac{\partial F_y}{\partial \dot{\phi}} \Delta \dot{\phi} + \frac{\partial F_y}{\partial \dot{\psi}} \Delta \dot{\psi}$$

$$\sum \partial F_y = \frac{\partial F_y}{\partial \beta} \beta + \frac{\partial F_y}{\partial \psi} \psi + \frac{\partial F_y}{\partial \phi} \phi + \frac{\partial F_y}{\partial \dot{\phi}} \dot{\phi} + \frac{\partial F_y}{\partial \dot{\psi}} \dot{\psi}$$

Initial value of ψ, β, ϕ are zero

$$\sum \partial F_y = \frac{\partial F_y}{\partial \beta} \beta + \frac{\partial F_y}{\partial \psi} \psi + \frac{\partial F_y}{\partial \phi} \phi + \frac{\partial F_y}{\partial \dot{\phi}} \dot{\phi} + \frac{\partial F_y}{\partial \dot{\psi}} \dot{\psi}$$

$$mU\dot{\beta} + mU\dot{\psi} = \frac{\partial F_y}{\partial \beta} \beta + \frac{\partial F_y}{\partial \psi} \psi + \frac{\partial F_y}{\partial \phi} \phi + \frac{\partial F_y}{\partial \dot{\phi}} \dot{\phi} + \frac{\partial F_y}{\partial \dot{\psi}} \dot{\psi}$$

$$\frac{mU}{Sq} \dot{\beta} + \frac{mU}{Sq} \dot{\psi} = \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\phi}} \dot{\phi} + \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\psi}} \dot{\psi}$$

$$\frac{mU}{Sq} \dot{\beta} + \frac{mU}{Sq} \dot{\psi} = \frac{1}{Sq} \frac{\partial F_y}{\partial \beta} \beta + \frac{1}{Sq} \frac{\partial F_y}{\partial \psi} \psi + \frac{1}{Sq} \frac{\partial F_y}{\partial \phi} \phi + \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\phi}} \dot{\phi} + \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\psi}} \dot{\psi}$$

$$\frac{mU}{Sq} \dot{\beta} + \frac{mU}{Sq} \dot{\psi} - \frac{1}{Sq} \frac{\partial F_y}{\partial \beta} \beta - \frac{1}{Sq} \frac{\partial F_y}{\partial \psi} \psi - \frac{1}{Sq} \frac{\partial F_y}{\partial \phi} \phi - \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\phi}} \dot{\phi} - \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\psi}} \dot{\psi} = \frac{F_{ya}}{Sq}$$

$$\frac{mU}{Sq} \dot{\beta} - \frac{1}{Sq} \frac{\partial F_y}{\partial \beta} \beta + \frac{mU}{Sq} \dot{\psi} - \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\psi}} \dot{\psi} - \frac{1}{Sq} \frac{\partial F_y}{\partial \psi} \psi - \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\phi}} \dot{\phi} - \frac{1}{Sq} \frac{\partial F_y}{\partial \phi} \phi = \frac{F_{ya}}{Sq}$$

$$\frac{mU}{Sq} \dot{\beta} - \frac{1}{Sq} \frac{\partial F_y}{\partial \beta} \beta + \left(\frac{mU}{Sq} - \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\psi}} \right) \dot{\psi} - \frac{1}{Sq} \frac{\partial F_y}{\partial \psi} \psi - \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\phi}} \dot{\phi} - \frac{1}{Sq} \frac{\partial F_y}{\partial \phi} \phi = \frac{F_{ya}}{Sq}$$

$$\frac{mU}{Sq} \dot{\beta} - \frac{1}{Sq} \frac{\partial F_y}{\partial \beta} \beta + \left(\frac{mU}{Sq} - \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\psi}} \right) \dot{\psi} - \frac{1}{Sq} \frac{\partial F_y}{\partial \psi} \psi - \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\phi}} \dot{\phi} - \frac{1}{Sq} \frac{\partial F_y}{\partial \phi} \phi = \frac{F_{ya}}{Sq}$$

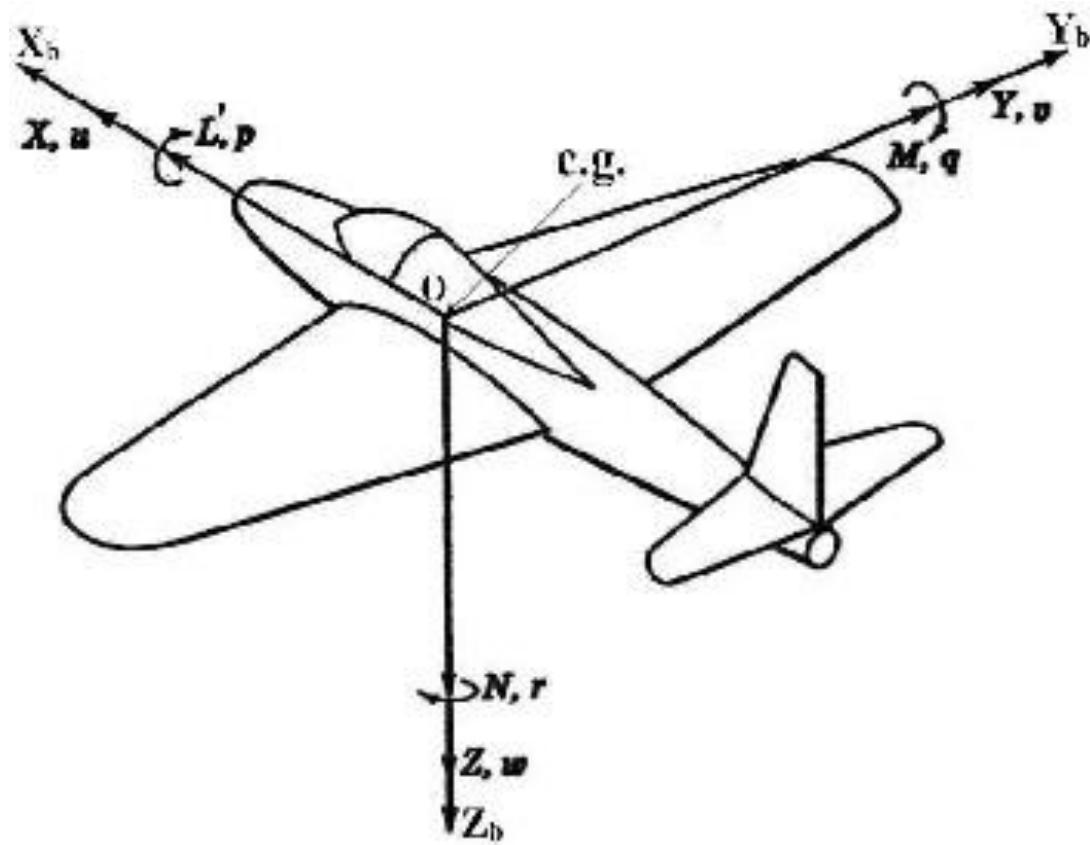
multiply and divide by $\frac{b}{2U}$

$$\frac{mU}{Sq} \dot{\beta} - \frac{1}{Sq} \frac{\partial F_y}{\partial \beta} \beta + \left(\frac{mU}{Sq} - \left(\frac{b}{2U} \right) \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\psi}} \left(\frac{2U}{b} \right) \right) \dot{\psi} - \frac{1}{Sq} \frac{\partial F_y}{\partial \psi} \psi - \frac{1}{Sq} \left(\frac{b}{2U} \right) \frac{\partial F_y}{\partial \dot{\phi}} \left(\frac{2U}{b} \right) \dot{\phi} - \frac{1}{Sq} \frac{\partial F_y}{\partial \phi} \phi = \frac{F_{ya}}{Sq}$$

$$\frac{mU}{Sq} \dot{\beta} - C_{y\beta} \beta + \left(\frac{mU}{Sq} - \frac{b}{2U} C_{yr} \right) \dot{\psi} - C_{y\psi} \psi - \frac{b}{2U} C_{yp} \dot{\phi} - C_{y\phi} \phi = \frac{F_{ya}}{Sq} = C_{ya}$$

$$C_{y\beta} = \frac{1}{Sq} \frac{\partial F_y}{\partial \beta} \quad C_{yr} = \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\psi}} \left(\frac{2U}{b} \right) \quad C_{y\psi} = \frac{1}{Sq} \frac{\partial F_y}{\partial \psi} \quad C_{yp} = \frac{1}{Sq} \frac{\partial F_y}{\partial \dot{\phi}} \left(\frac{2U}{b} \right) \quad C_{y\phi} = \frac{1}{Sq} \frac{\partial F_y}{\partial \phi}$$

MOMENT EQUATION ALONG ROLL AXIS



$$\sum \Delta L = \dot{p}I_x - \dot{r}J_{xz}$$

$$\Sigma \partial L = \frac{\partial L}{\partial \beta} d\beta + \frac{\partial L}{\partial \psi} d\psi + \frac{\partial L}{\partial \phi} d\phi + \frac{\partial L}{\partial \dot{\phi}} \partial \dot{\phi} + \frac{\partial L}{\partial \dot{\psi}} \partial \dot{\psi}$$

$$\Sigma \Delta L = \frac{\partial L}{\partial \beta} \Delta \beta + \frac{\partial L}{\partial \psi} \Delta \psi + \frac{\partial L}{\partial \phi} \Delta \phi + \frac{\partial L}{\partial \dot{\phi}} \Delta \dot{\phi} + \frac{\partial L}{\partial \dot{\psi}} \Delta \dot{\psi}$$

$$\Sigma \partial L = \frac{\partial L}{\partial \beta} \beta + \frac{\partial L}{\partial \psi} \psi + \frac{\partial L}{\partial \phi} \phi + \frac{\partial L}{\partial \dot{\phi}} \dot{\phi} + \frac{\partial L}{\partial \dot{\psi}} \dot{\psi}$$

$$\ddot{\phi}I_x - \ddot{\psi}J_{xz} = \frac{\partial L}{\partial \beta} \beta + \frac{\partial L}{\partial \psi} \psi + \frac{\partial L}{\partial \phi} \phi + \frac{\partial L}{\partial \dot{\phi}} \dot{\phi} + \frac{\partial L}{\partial \dot{\psi}} \dot{\psi}$$

Derivatives of L and N with respect to ϕ and ψ are zero

$$\ddot{\phi}I_x - \ddot{\psi}J_{xz} = \frac{\partial L}{\partial \beta} \beta + \frac{\partial L}{\partial \phi} \dot{\phi} + \frac{\partial L}{\partial \dot{\psi}} \dot{\psi}$$

$$\ddot{\phi}I_x - \ddot{\psi}J_{xz} - \frac{\partial L}{\partial \beta} \beta - \frac{\partial L}{\partial \dot{\phi}} \dot{\phi} - \frac{\partial L}{\partial \dot{\psi}} \dot{\psi} = L_a$$

$$\ddot{\phi}I_x - \frac{\partial L}{\partial \dot{\phi}} \dot{\phi} - \ddot{\psi}J_{xz} - \frac{\partial L}{\partial \dot{\psi}} \dot{\psi} - \frac{\partial L}{\partial \beta} \beta = L_a$$

$$\div Sqb \quad \frac{I_x}{Sq b} \ddot{\phi} - \frac{1}{Sq b} \frac{\partial L}{\partial \dot{\phi}} \dot{\phi} - \frac{J_{xz}}{Sq b} \ddot{\psi} - \frac{1}{Sq b} \frac{\partial L}{\partial \dot{\psi}} \dot{\psi} - \frac{1}{Sq b} \frac{\partial L}{\partial \beta} \beta = \frac{L_a}{Sq b} = C_{la}$$

multiply and divide by $\frac{b}{2U}$

$$\frac{I_x}{Sq b} \ddot{\phi} - \frac{1}{Sq b} \left(\frac{b}{2U} \right) \frac{\partial L}{\partial \dot{\phi}} \left(\frac{2U}{b} \right) \dot{\phi} - \frac{J_{xz}}{Sq b} \ddot{\psi} - \frac{1}{Sq b} \left(\frac{b}{2U} \right) \frac{\partial L}{\partial \dot{\psi}} \left(\frac{2U}{b} \right) \dot{\psi} - \frac{1}{Sq b} \frac{\partial L}{\partial \beta} \beta = \frac{L_a}{Sq b} = C_{la}$$

$$\frac{I_x}{Sq b} \ddot{\phi} - \left(\frac{b}{2U} \right) Cl_p \dot{\phi} - \frac{J_{xz}}{Sq b} \ddot{\psi} - \left(\frac{b}{2U} \right) Cl_r \dot{\psi} - Cl_\beta \beta = \frac{L_a}{Sq b} = C_{la}$$

$$Cl_p = \frac{1}{Sq b} \frac{\partial L}{\partial \dot{\phi}} \left(\frac{2U}{b} \right) \quad Cl_r = \frac{1}{Sq b} \frac{\partial L}{\partial \dot{\psi}} \left(\frac{2U}{b} \right) \quad Cl_\beta = \frac{1}{Sq b} \frac{\partial L}{\partial \beta}$$

$$\Sigma \partial N = \frac{\partial N}{\partial \beta} d\beta + \frac{\partial N}{\partial \psi} d\psi + \frac{\partial N}{\partial \phi} d\phi + \frac{\partial N}{\partial \dot{\phi}} \partial \dot{\phi} + \frac{\partial N}{\partial \dot{\psi}} \partial \dot{\psi}$$

$$\Sigma \partial N = \frac{\partial N}{\partial \beta} \Delta \beta + \frac{\partial N}{\partial \psi} \Delta \psi + \frac{\partial N}{\partial \phi} \Delta \phi + \frac{\partial N}{\partial \dot{\phi}} \Delta \dot{\phi} + \frac{\partial N}{\partial \dot{\psi}} \Delta \dot{\psi}$$

$$\Sigma \partial N = \frac{\partial N}{\partial \beta} \beta + \frac{\partial N}{\partial \psi} \psi + \frac{\partial N}{\partial \phi} \phi + \frac{\partial N}{\partial \dot{\phi}} \dot{\phi} + \frac{\partial N}{\partial \dot{\psi}} \dot{\psi}$$

MOMENT EQUATION ALONG YAW AXIS

$$\sum \Delta N = \ddot{\psi} I_z - \ddot{\phi} J_{xz}$$

$$\ddot{\psi} I_z - \ddot{\phi} J_{xz} = \frac{\partial N}{\partial \beta} \beta + \frac{\partial N}{\partial \psi} \psi + \frac{\partial N}{\partial \phi} \phi + \frac{\partial N}{\partial \dot{\phi}} \dot{\phi} + \frac{\partial N}{\partial \dot{\psi}} \dot{\psi}$$

$$\ddot{\psi} I_z - \ddot{\phi} J_{xz} - \frac{\partial N}{\partial \beta} \beta - \frac{\partial N}{\partial \psi} \psi - \frac{\partial N}{\partial \phi} \phi - \frac{\partial N}{\partial \dot{\phi}} \dot{\phi} - \frac{\partial N}{\partial \dot{\psi}} \dot{\psi} = N_a$$

Derivatives of L and N with respect to ϕ and ψ are zero

$$\ddot{\psi} I_z - \ddot{\phi} J_{xz} - \frac{\partial N}{\partial \beta} \beta - \frac{\partial N}{\partial \dot{\phi}} \dot{\phi} - \frac{\partial N}{\partial \dot{\psi}} \dot{\psi} = N_a$$

$$-\ddot{\phi} J_{xz} - \frac{\partial N}{\partial \dot{\phi}} \dot{\phi} + \ddot{\psi} I_z - \frac{\partial N}{\partial \dot{\psi}} \dot{\psi} - \frac{\partial N}{\partial \beta} \beta = N_a$$

$$\div Sqb$$

$$-\frac{J_{xz}}{Sq b}\ddot{\phi} - \frac{1}{Sq b}\frac{\partial N}{\partial \dot{\phi}}\dot{\phi} + \frac{I_z}{Sq b}\ddot{\psi} - \frac{1}{Sq b}\frac{\partial N}{\partial \dot{\psi}}\dot{\psi} - \frac{1}{Sq b}\frac{\partial N}{\partial \beta}\beta = \frac{N_a}{Sq b} = C_{na}$$

$$\frac{b}{2U}$$

$$-\frac{J_{xz}}{Sq b}\ddot{\phi} - \frac{1}{Sq b}\left(\frac{b}{2U}\right)\frac{\partial N}{\partial \dot{\phi}}\left(\frac{2U}{b}\right)\dot{\phi} + \frac{I_z}{Sq b}\ddot{\psi} - \frac{1}{Sq b}\left(\frac{b}{2U}\right)\frac{\partial N}{\partial \dot{\psi}}\left(\frac{2U}{b}\right)\dot{\psi} - \frac{1}{Sq b}\frac{\partial N}{\partial \beta}\beta = \frac{N_a}{Sq b} = C_{na}$$

$$-\frac{J_{xz}}{Sq b}\ddot{\phi} - \frac{b}{2U}C_{np}\dot{\phi} + \frac{I_z}{Sq b}\ddot{\psi} - \frac{b}{2U}C_{nr}\dot{\psi} - C_{n\beta}\beta = C_{na}$$

$$C_{np} = \frac{1}{Sq b}\left(\frac{2U}{b}\right)\frac{\partial N}{\partial \dot{\phi}}$$

$$C_{nr} = \frac{1}{Sq b}\left(\frac{2U}{b}\right)\frac{\partial N}{\partial \dot{\psi}}$$

$$C_{n\beta} = \frac{1}{Sq b}\frac{\partial N}{\partial \beta}$$

$$\frac{I_x}{Sq b} \ddot{\phi} - \left(\frac{b}{2U} \right) Cl_p \dot{\phi} - \frac{J_{xz}}{Sq b} \ddot{\psi} - \left(\frac{b}{2U} \right) Cl_r \dot{\psi} - Cl_\beta \beta = \frac{L_a}{Sq b} = C_{la}$$

$$-\frac{J_{xz}}{Sq b} \ddot{\phi} - \frac{b}{2U} C_{np} \dot{\phi} + \frac{I_z}{Sq b} \ddot{\psi} - \frac{b}{2U} C_{nr} \dot{\psi} - C_{n\beta} \beta = C_{na}$$

$$\frac{mU}{Sq} \dot{\beta} - C_{y\beta} \beta + \left(\frac{mU}{Sq} - \frac{b}{2U} C_{yr} \right) \dot{\psi} - C_{y\psi} \psi - \frac{b}{2U} C_{yp} \dot{\phi} - C_{y\phi} \phi = \frac{F_{ya}}{Sq} = C_{ya}$$

rearrange the last equation

$$-\frac{b}{2U} C_{yp} \dot{\phi} - C_{y\phi} \phi + \left(\frac{mU}{Sq} - \frac{b}{2U} C_{yr} \right) \dot{\psi} - C_{y\psi} \psi + \frac{mU}{Sq} \dot{\beta} - C_{y\beta} \beta = \frac{F_{ya}}{Sq} = C_{ya}$$

$$-\frac{b}{2U} C_{yp} \dot{\phi} - C_{y\phi} \phi + \left(\frac{mU}{Sq} - \frac{b}{2U} C_{yr} \right) \dot{\psi} - C_{y\psi} \psi + \left(\frac{mU}{Sq} s - C_{y\beta} \right) \beta = \frac{F_{ya}}{Sq} = C_{ya}$$

$C_{yp}, C_{yr} = 0$ (vertical Tail may neglected)

$$-C_{y\phi} \phi + \left(\frac{mU}{Sq} \right) \dot{\psi} - C_{y\psi} \psi + \left(\frac{mU}{Sq} s - C_{y\beta} \right) \beta = \frac{F_{ya}}{Sq} = C_{ya}$$

$$-C_{y\phi} \phi + \left(\frac{mU}{Sq} \dot{\psi} - C_{y\psi} \psi \right) + \left(\frac{mU}{Sq} s - C_{y\beta} \right) \beta = \frac{F_{ya}}{Sq} = C_{ya}$$

Symbol	Definition	Origin	Equation	Typical Values
C_{I_β}	$\frac{1}{Sqb} \frac{\partial \mathcal{L}}{\partial \beta}$	Dihedral and vertical tail	Ref. 1, Chapter 9 Ref. 2, Section 3.10	-0.06
C_{I_p}	$\frac{1}{Sqb} \left(\frac{2U}{b} \right) \frac{\partial \mathcal{L}}{\partial p}$	Wing damping	Ref. 1, Chapter 9	-0.4
C_{I_r}	$\frac{1}{Sqb} \left(\frac{2U}{b} \right) \frac{\partial \mathcal{L}}{\partial r}$	Differential wing normal force	$\frac{C_L^w}{4}$	0.06
C_{n_β}	$\frac{1}{Sqb} \frac{\partial \mathcal{N}}{\partial \beta}$	Directional stability	Ref. 1, Chapter 8 Ref. 2, Section 3.9	0.11
C_{n_p}	$\frac{1}{Sqb} \left(\frac{2U}{b} \right) \frac{\partial \mathcal{N}}{\partial p}$	Differential wing chord force	$-\frac{C_L^w}{8} \left(1 - \frac{d\epsilon}{d\alpha} \right)$	-0.015
C_{n_r}	$\frac{1}{Sqb} \left(\frac{2U}{b} \right) \frac{\partial \mathcal{N}}{\partial r}$	Damping in yaw	$-\frac{C_D^w}{4} - 2\eta_v \frac{S_v}{S} \left(\frac{I_v}{b} \right)^2 \left(\frac{dC_L}{d\alpha} \right)^v$	-0.12
C_{y_β}	$\frac{1}{Sq} \frac{\partial F_y}{\partial \beta}$	Fuselage and vertical tail	No simple equation	-0.6
C_{y_ϕ}	$\frac{1}{Sq} \frac{\partial F_y}{\partial \phi}$	Gravity	$\frac{mg}{Sq} \cos \Theta$	
C_{y_p}	$\frac{1}{Sq} \left(\frac{2U}{b} \right) \frac{\partial F_y}{\partial p}$	Vertical tail	Neglect	
C_{y_ψ}	$\frac{1}{Sq} \left(\frac{\partial F_y}{\partial \psi} \right)$	Gravity	$\frac{mg}{Sq} \sin \Theta$	
C_{y_r}	$\frac{1}{Sq} \left(\frac{2U}{b} \right) \frac{\partial F_y}{\partial r}$	Vertical tail	Neglect	

SOLUTION FOR LATERAL EQUATION

$$\left(\frac{I_x}{Sq b} s^2 - \frac{b}{2U} C_{lp} s \right) \phi(s) + \left(-\frac{J_{xz}}{Sq b} s^2 - \frac{b}{2U} C_{lr} s \right) \psi(s) - C_{l\beta} \beta(s) = 0$$

$$-\left(\frac{J_{xz}}{Sq b} s^2 - \frac{b}{2U} C_{np} s \right) \phi(s) + \left(\frac{I_z}{Sq b} s^2 - \frac{b}{2U} C_{nr} s \right) \psi(s) - C_{n\beta} \beta(s) = 0$$

$$-C_{y\phi}(s) + \left(\frac{mU}{Sq} s - C_{y\psi} \right) \psi(s) + \left(\frac{mU}{Sq} s - C_{y\beta} \right) \beta(s) = 0$$

$$\Theta = 0$$

$$Mach = 0.394$$

$$m = 5900 \text{ slugs}$$

$$U = 440 \text{ ft/sec}$$

$$S = 2400 \text{ sq ft}$$

$$I_x = 1.955 \times 10^6 \text{ slug ft}^2$$

$$I_z = 4.2 \times 10^6 \text{ slug ft}^2$$

$$J_{xz} = 0 \text{ (by assumption)}$$

$$Cl_p = -0.38$$

$$C_{lr} = \frac{CL}{4} = \frac{0.344}{4} = 0.086$$

$$b = 130 \text{ ft}$$

$$C_{np} = -0.0228$$

$$C_{n\beta} = 0.096$$

$$C_{nr} = -0.107$$

$$C_{y\beta} \simeq -0.6$$

$$C_{y\phi} = \frac{mg}{Sq} = CL = 0.344$$

$$C_{y\psi} = 0$$

$$C_{l\beta} = -0.057$$

$$\frac{b}{2U} = \frac{130}{2(440)} = 0.148 \text{ sec}$$

$$\frac{b}{2U} C_{lp} = 0.0553 \text{ sec}$$

$$\frac{b}{2U} C_{lr} = 0.0128 \text{ sec}$$

$$\frac{b}{2U} C_{np} = -0.00338 \text{ sec}$$

$$\frac{b}{2U} C_{nr} = -0.0158 \text{ sec}$$

$$q = \frac{\rho}{2} U^2 = \frac{(0.002378)(440)^2}{2} = 230 \text{ lb/sq ft}$$

$$\frac{I_x}{Sq b} = \frac{1.955 \times 10^6}{(2400)(230)(130)} = 0.02725 \text{ sec}^2$$

$$\frac{I_z}{Sq b} = \frac{4.2 \times 10^6}{(2400)(230)(130)} = 0.0585 \text{ sec}^2$$

$$\frac{mU}{Sq} = \frac{(5900)(440)}{(2400)(230)} = 4.7 \text{ sec}$$

$$\begin{bmatrix} 0.02725s^2 + 0.0553s & -0.0128s & 0.057 \\ 0.00338s & 0.058s^2 + 0.0158s & -0.096 \\ -0.344 & 4.71s & 4.71s + 006 \end{bmatrix} = 0$$

$$0.00748s^5 + 0.01827s^4 + 0.01876s^3 + 0.0275s^2 - 0.0001135s = 0$$

Dividing by the s^5 coefficient

$$s^5 + 2.44s^4 + 2.51s^3 + 3.68s^2 - 0.0152s = 0$$

$$s(s^2 + 0.380s + 1.813)(s + 2.09)(s - 0.004) = 0$$

$$\underbrace{s(s^2 + 0.380s + 1.813)}_{Dutchroll} \underbrace{(s + 2.09)}_{rollout} \underbrace{(s - 0.004)}_{Spiral} = 0$$

$$(s^2 + 2\delta\omega_p s + \omega_p^2) = (s^2 + 0.380s + 1.813) \quad (s + 2.09) \quad s - 0.004$$

$$\omega_D = 1.345 \text{ rad/sec}$$

$$\delta_D = 0.14$$

$$\tau_{rollout} = \frac{1}{2.09} = 0.478 \quad \tau_{spiral} = -\frac{1}{0.004} = 250 \text{ sec}$$

THANK YOU